DQ-003-1016002 Seat No. ______

B. Sc. (Sem. VI) (CBCS) Examination

April - 2022

Mathematics : Paper - M - 09 (A)

(Mathematical Analysis - II & Abstract Algebra - II)

(Old Course)

Faculty Code : 003
Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Write answer of each question in your main answer sheet.

1 (a) Answer the following questions briefly : **4**
(1) Define Open Cover.
(2) Define : Connected set.
(3) Determine whether the subset $\{0,1\}$ of metric space R is compact or not.
(4) Define compact set.

(b) Attempt any one out of two : **2**
(1) Show that subset $R-\{3\}$ is not connected, where $a \in R$.
(2) If A and B are compact sets of metric space X then prove that $A \cap B$ is also compact.

(c) Attempt any one out of two : **3**
(1) State and prove Heine-Borel theorem.
(2) If F is a closed subset of metric space X and K is a compact subset of X. Then prove that $F \cap K$ is also compact.

(d) Attempt any one out of two : **5**
(1) State and prove theorem of nested intervals.
(2) Prove that continuous image of compact set is compact.

2 (a) Answer the following questions briefly : **4**

(1) Define Laplace transform.

(2) Find $L^{-1}\left(\frac{1}{s-2}\right)$

(3) Find $L^{-1}\left(\frac{1}{s^2-4}\right)$

(4) Show that $L(1)=\frac{1}{s}$, where $s>0$.

(b) Attempt any **one** out of two : **2**

(1) Find $L^{-1}\left(\frac{2s+6}{s^2+4}\right)$

(2) Find $L\left(\frac{e^{at}-1}{a}\right)$, where a is constant.

(c) Attempt any one out of two : **3**

(1) Find Laplace transform of $e^{-2t}\sin^2 t$.

(2) If $L\{f(t)\}=\overline{f}(s)$ then prove that

$L\{e^{at}f(t)\}=\overline{f}(s-a)$

(d) Attempt any one out of two : **5**

(1) If $f(t)=e^t,\ t\le 2$

$=3,\ t>2$ then find $L\{f(t)\}$.

(2) Prove that $L^{-1}\left(\frac{s^3}{s^4-a^4}\right)=\frac{1}{2}(\cos at+\cosh at)$

3 (a) Answer the following questions briefly : **4**

(1) Find $L(t\sin t)$

(2) Write convolution theorem.

(3) Find $L(t\sinh at)$

(4) Find $L\left(\frac{\sin t}{t}\right)$

(b) Attempt any one out of two : **2**

(1) If $L\{f(t)\} = \overline{f}(s)$ then prove

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\overline{f}(s)\right]$$

(2) If $L\{f(t)\} = \overline{f}(s)$ then prove $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f}(s)\,ds$.

(c) Attempt any one out of two : **3**

(1) Prove that $L\{te^{-t}\sin t\} = \left(\frac{2(s+1)}{\left(s^2+2s+2\right)^2}\right)$

(2) Prove that $L^{-1}\left(\log\left(\frac{s+b}{s+a}\right)\right) = \frac{e^{-at} - e^{-bt}}{t}$

(d) Attempt any one out of two : **5**

(1) Prove that $L^{-1}\left\{\frac{1}{\left(s^2+a^2\right)^2}\right\} = \frac{1}{2a^3}(\sin at - at\cos at)$

(2) Using convolution theorem, prove

$$L^{-1}\left\{\frac{1}{s\left(s^2+4\right)}\right\} = \frac{1}{4}(1-\cos 2t)$$

4 (a) Answer the following questions briefly : **4**

(1) Define Subring

(2) Define Homomorphism of Groups

(3) Define Natural mapping

(4) Define Kernel of homomorphism

(b) Attempt any one out of two : **2**

(1) Let $\varnothing : (G, *) \to (G', \Delta)$ is Homomorphism. If $H \leq G$ then prove $\varnothing(H) \leq G'$.

(2) If $\varnothing : (G, *) \to (G', \Delta)$ is a Homomorphism. Then $\varnothing(e) = e'$ where e and e' are identity elements of G and G' respectively.

(c) Attempt any one out of two : **3**

(1) Prove that a cyclic group of order eight is homomorphism to a cyclic group of order four.

(2) If G is a cyclic group of prime order then prove that a homomorphism $\varnothing : G \to G$ is either an isomorphism or $\varnothing(a) = e;\ \forall a \in G$

(d) Attempt any one out of two : **5**

(1) State and prove first fundamental theorem of homomorphism.

(2) Prove that A homomorphism $\varnothing : (G, *) \to (G', \Delta)$ is one-of iff $k_{\varnothing} = \{e\}$.

5 (a) Answer the following questions briefly : **4**

(1) Define constant polynomial

(2) If polynomial $f = \{0, 3, 2, 7, 0, 0, 0, 0, ...\}$ then find order of f.

(3) Define Linear polynomial.

(4) Define Monic polynomial.

(b) Attempt any one out of two : **2**

(1) Find conjugate of quaternion $1 - 3i + 2j - k$.

(2) If $f(x) = (1, 3, 2, 2, 0, 0......)$ and $g(x) = (2, 2, 0, 0, 3, 0.......) \in R[x]$ then find $f(x) + g(x)$.

(c) Attempt any one out of two : **3**

(1) State and prove Remainder theorem of polynomails.

(2) Find g.c.d. of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and $g(x) = 2x^2 - 3x + 5 \in R[X]$ and express it in the form $a(x) f(x) + b(x) g(x)$.

(d) Attempt any one out of two : **5**

(1) State and prove division algorithm for polynomials.

(2) Prove that any ideal in integration domain $F[X]$ is a principal ideal.